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Propagator of an electron in a cavity in the presence of a coherent photonic field

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Abstract. In the present paper we consider the case of an electron under the presence of a single mode field in a cavity linearly polarized in the z-direction. We adopt the dipole approximation and we derive the full propagator of the electron. In the present case we suppose that the field is in a coherent state. The parameters of the propagator involve Mathieu functions. Finally we extract the time evolution of an initially Gaussian wavepacket and its probability density. The present theory is applicable to the interaction of strong fields with atoms.

PACS. 03.65.Fd Algebraic methods

1 Introduction

The study of cavity effects has been a main area of research for a long time. It allows the production and use of certain radiation states and the study of their interaction with atoms.

In previous work [1,2], we have studied a specific cavity effect. We have considered a hydrogen atom interacting with radiation and we have studied the transitions induced by the field for initial and final field states such as vacuum, coherent and squeezed states. In such problems the standard approach we use is the elimination of the field variables using the coherent states path integral formalism [3–11], and then handling the path integral over the electron's coordinate with either exact methods or numerical ones (*e.g.* Monte Carlo), depending on the specific problem and its geometry [12–15].

In the present paper we study quantum mechanically an electron in a cavity, under the action of a single mode field polarized in the z-direction. In fact the present scheme is an alternative form of the standard Jaynes-Cummings model which has been worked to exhaustion of its possibilities [16].

The present paper proceeds in the following order. In Section 2 we give the system Hamiltonian and the propagator integrated over the photonic field in a coherent state representation. Then we calculate the reduced propagator exactly supposing initial and final coherent states. Finally we apply the results of [17] to find the propagator. Mathieu function appears on integrating the path integral over the electron coordinates. In Section 3 we apply the results of the previous section to the evolution of a Gaussian wavepacket. In Section 4 we give our conclusions.

2 System Hamiltonian and propagator of the electron

We now consider the system Hamiltonian H. It can be written as a sum of three terms. The free electron Hamiltonian H_{e} in its center of mass system, the single mode field one H_{f} and the interaction term H_{I}

$$H = H_{\mathrm{e}} + H_{\mathrm{f}} + H_{\mathrm{I}}. \tag{1}$$

Particularly the Hamiltonian of the electron is given as

$$H_{\mathrm{e}} = \frac{\mathbf{p}^2}{2} \tag{2}$$

and the Hamiltonian of the cavity mode has the form

$$H_{\mathrm{f}} = \omega\left(a^+a + \frac{1}{2}\right)\cdot \tag{3}$$

In the present paper we suppose that only one mode is present in the cavity.

Finally, the interaction Hamiltonian in the length form is given as

$$H_{\mathrm{I}} = -e\mathbf{r}\cdot\mathbf{E}_{\mathrm{f}}. \tag{4}$$

The second quantized form of the field operator is given as

$$\mathbf{E}_{\mathrm{f}}(\mathbf{r}) = \frac{1}{\sqrt{V}}\mathrm{i}l(\omega)\hat{\varepsilon}\left[a\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} - a^+\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\right] \tag{5}$$

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where V is the cavity volume and $l(\omega)$ is a real function of frequency given as $l(\omega)=\sqrt{\hbar\omega/2\varepsilon_0}$.

In (4) we use the dipole approximation $\left(\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\approx 1\right)$ supposing that the electron wavepacket is not very much spread in space in relation to the radiation wavelength, so that the field operator has the form

$$\mathbf{E}_{\mathrm{f}}=\frac{1}{\sqrt{V}}\mathrm{i}l(\omega)\hat{\varepsilon}\left(a-a^{+}\right) \tag{6}$$

and H_{I} takes the form

$$H_{\mathrm{I}}=-\frac{1}{\sqrt{V}}\mathrm{i}el(\omega)\hat{\varepsilon}\cdot\mathbf{r}(t)\left(a-a^{+}\right). \tag{7}$$

Now we combine the terms (3, 7) involving field variables in the term

$$H_0(a^{+},a;t)=H_{\mathrm{f}}+H_{\mathrm{I}}=\omega\left(a^{+}a+\frac{1}{2}\right)+g(t)a+g^{*}(t)a^{+} \tag{8}$$

where

$$g(t)=-\frac{1}{\sqrt{V}}\mathrm{i}el(\omega)\hat{\varepsilon}\cdot\mathbf{r}(t). \tag{9}$$

The propagator corresponding to (8) has be derived *via* path integral methods [18].

In fact on considering the Hamiltonian (1) the full propagator can be obtained by integrating over both the space and photonic field variables. At first we integrate over the field variables, which appear only in H_0. There results a path integral of only the spatial variables. It is given by the following expression up to an overall phase coming from the constant in (3)

$$K(\alpha_{\mathrm{f}},\mathbf{r}_{\mathrm{f}};\alpha_{\mathrm{i}},\mathbf{r}_{\mathrm{i}};t)=\int_{\mathbf{r}(0)=\mathbf{r}_{\mathrm{i}}}^{\mathbf{r}(t)=\mathbf{r}_{\mathrm{f}}}D\mathbf{r}(\tau)$$
$$\times\exp\left[\begin{array}{l}\mathrm{i}\displaystyle\int_0^t\mathrm{d}\tau\frac{\dot{\mathbf{r}}^2(\tau)}{2}-\mathrm{i}\int_0^t\mathrm{d}\tau g(\tau)Z(\tau)-\frac{1}{2}\left(|\alpha_{\mathrm{f}}|^2+|\alpha_{\mathrm{i}}|^2\right)\\ +Y(t)\alpha_{\mathrm{f}}^{*}\alpha_{\mathrm{i}}+Z(t)\alpha_{\mathrm{f}}^{*}-\mathrm{i}\alpha_{\mathrm{i}}X(t)\end{array}\right]. \tag{10}$$

$Y(t)$, $X(t)$ and $Z(t)$ are given as

$$Y(t)=\exp\left[-\mathrm{i}\int_0^t\mathrm{d}\tau\omega(\tau)\right]=\exp(-\mathrm{i}\omega t), \tag{11}$$

$$X(t)=\int_0^t\mathrm{d}\tau g(\tau)Y(\tau), \tag{12}$$

$$Z(t)=-\mathrm{i}\int_0^t\mathrm{d}\tau g^{*}(\tau)\exp\left[-\mathrm{i}\int_\tau^t\mathrm{d}\tau'\omega(\tau')\right]. \tag{13}$$

The propagator (10) with diagonal photonic field variables can be written as

$$K(\alpha,\mathbf{r}_{\mathrm{f}};\alpha,\mathbf{r}_{\mathrm{i}};t)=\int_{\mathbf{r}(0)=\mathbf{r}_{\mathrm{i}}}^{\mathbf{r}(t)=\mathbf{r}_{\mathrm{f}}}D\mathbf{r}(\tau)$$
$$\times\exp\left[\mathrm{i}\int_0^t\mathrm{d}\tau\frac{\dot{\mathbf{r}}^2(\tau)}{2}+A-B|\alpha|^2+D_1\alpha+D\alpha^{*}\right] \tag{14}$$

where after using (9), the parameters are given as

$$A(t)=-\frac{1}{V}e^2l^2(\omega)\int_0^t\mathrm{d}\tau\int_0^\tau\mathrm{d}\rho\hat{\varepsilon}\cdot\mathbf{r}(\tau)\hat{\varepsilon}\cdot\mathbf{r}(\rho)\mathrm{e}^{\mathrm{i}\omega(\rho-\tau)}, \tag{15}$$

$$B(t)=1-Y(t)=1-\exp(-\mathrm{i}\omega t), \tag{16}$$

$$D(t)=\frac{1}{\sqrt{V}}el(\omega)\int_0^t\mathrm{d}\tau\hat{\varepsilon}\cdot\mathbf{r}(\tau)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{e}^{-\mathrm{i}\omega t}, \tag{17}$$

$$D_1(t)=-\frac{1}{\sqrt{V}}el(\omega)\int_0^t\mathrm{d}\tau\hat{\varepsilon}\cdot\mathbf{r}(\tau)\mathrm{e}^{\mathrm{i}\omega\tau}. \tag{18}$$

In this paper we consider that we have a field transition from a coherent state to a second one. Other kind of photonic transitions can be treated similarly.

Now we can integrate the propagator (14) over the field variable α between a final $|\gamma\rangle$ and an initial $|\beta\rangle$ coherent field state to obtain the following reduced propagator for the motion of the electron

$$\widetilde{K}^{\gamma,\beta}(\mathbf{r}_{\mathrm{f}},\mathbf{r}_{\mathrm{i}};t)=C(t)\int_{\mathbf{r}(0)=\mathbf{r}_{\mathrm{i}}}^{\mathbf{r}(t)=\mathbf{r}_{\mathrm{f}}}D\mathbf{r}(\rho)\exp\left\{\mathrm{i}S_{\mathrm{tot}}\left[\mathbf{r}\right]\right\} \tag{19}$$

where

$$C(t)=\frac{\exp\left(\dfrac{\beta\gamma^{*}}{B(t)}-\dfrac{1}{2}|\beta|^2-\dfrac{1}{2}|\gamma|^2\right)}{B(t)} \tag{20}$$

$$S_{\mathrm{tot}}\left[\mathbf{r}\right]=\int_0^t\left\{\frac{\dot{\mathbf{r}}^2(\rho)}{2}\right.$$
$$\left.-\frac{1}{\sqrt{V}}el(\omega)\left(\begin{array}{l}\dfrac{\mathrm{i}}{\exp(\mathrm{i}\omega t)-1}\gamma^{*}\mathrm{e}^{\mathrm{i}\omega\rho}\\ +\dfrac{\mathrm{i}}{\exp(-\mathrm{i}\omega t)-1}\beta\mathrm{e}^{-\mathrm{i}\omega\rho}\end{array}\right)\hat{\varepsilon}\cdot\mathbf{r}(\rho)\right\}\mathrm{d}\rho$$
$$+\frac{1}{V}e^2l^2(\omega)\int_0^t\mathrm{d}\rho\int_0^\rho\mathrm{d}\sigma\hat{\varepsilon}\cdot\mathbf{r}(\rho)\hat{\varepsilon}\cdot\mathbf{r}(\sigma)\xi(t,\rho-\sigma) \tag{21}$$

and the function $\xi(t,\rho-\sigma)$ is given as

$$\xi(t,\rho-\sigma)=\mathrm{i}\mathrm{e}^{-\mathrm{i}\omega(\rho-\sigma)}+\mathrm{i}\frac{2}{\mathrm{e}^{\mathrm{i}\omega t}-1}\cos[\omega(\rho-\sigma)]. \tag{22}$$

Now we proceed to a certain approximation of the exact action (21). On performing the change of variables $\tau=\rho-\sigma$ one has the integral

$$\int_0^\rho\mathrm{d}\sigma\hat{\varepsilon}\cdot\mathbf{r}(\sigma)\xi(t,\rho-\sigma)=\int_0^\rho\mathrm{d}\tau\xi(t,\tau)\hat{\varepsilon}\cdot\mathbf{r}(\rho-\tau). \tag{23}$$

Now we perform the Taylor expansion

$$\hat{\varepsilon}\cdot\mathbf{r}(\rho-\tau)=\hat{\varepsilon}\cdot\mathbf{r}(\rho)-\tau\hat{\varepsilon}\cdot\dot{\mathbf{r}}(\rho)+\ldots \tag{24}$$

Then after substitution of the Taylor expansion in the integral (23) the first term $\mathbf{r}(\rho)$ can be taken out of the integral and the following integral arises

$$\nu(t,\rho)=\int_0^\rho\xi(t,\rho-\sigma)\mathrm{d}\sigma$$
$$=\frac{1}{\omega}\left[1+\csc\left(\frac{\omega t}{2}\right)\sin\left(\omega\rho-\frac{\omega t}{2}\right)\right] \tag{25}$$

and we obtain expression (27) (see below). Higher order terms in the Taylor expansion are in fact negligible as they are going to involve powers of $S = el(\omega)/\sqrt{V}$ and their contribution in the action is going to be of order S^3 and on, as the last term in the action (21) already involve an S^2 contribution. This fact can be shown by taking into account the equation of motion that can be derived from the action (21) *via* standard methods. In fact it reads as

$$\ddot{\mathbf{r}}(\rho) - 2\frac{1}{V}e^2 l^2(\omega)\hat{\varepsilon}\int_0^\rho \mathrm{d}\sigma\hat{\varepsilon}\cdot\mathbf{r}(\sigma)\xi(t,\rho-\sigma) + \frac{1}{\sqrt{V}}el(\omega)\hat{\varepsilon}\left[\frac{\mathrm{i}}{\exp(\mathrm{i}\omega t)-1}\gamma^*\mathrm{e}^{\mathrm{i}\omega\rho} + \frac{\mathrm{i}}{\exp(-\mathrm{i}\omega t)-1}\beta\mathrm{e}^{-\mathrm{i}\omega\rho}\right] = 0. \quad (26)$$

Powers of S in fact involve powers of the fine structure constant as well as the volume and their neglectfulness to a first order approximation is a standard method of approach in QED.

Finally the action (21) becomes

$$S_{\mathrm{tot}}[\mathbf{r}] = \int_0^t \left\{\frac{\dot{\mathbf{r}}^2(\rho)}{2} - \frac{1}{\sqrt{V}}el(\omega)\left(\frac{\mathrm{i}}{\exp(\mathrm{i}\omega t)-1}\gamma^*\mathrm{e}^{\mathrm{i}\omega\rho} + \frac{\mathrm{i}}{\exp(-\mathrm{i}\omega t)-1}\beta\mathrm{e}^{-\mathrm{i}\omega\rho}\right)\hat{\varepsilon}\cdot\mathbf{r}(\rho)\right\}\mathrm{d}\rho + \frac{1}{V}e^2l^2(\omega)\int_0^t \mathrm{d}\rho\left(\hat{\varepsilon}\cdot\mathbf{r}(\rho)\right)^2\nu(t,\rho). \quad (27)$$

Supposing a linear polarization on the z-direction we have three uncoupled one-dimensional problems. The propagator in the z-direction corresponds to a general driven time-dependent oscillator. On applying the work of Lo [17] it is given as

$$K_z^{\gamma,\beta}(z_\mathrm{f},z_\mathrm{i},t) = \sqrt{\frac{\mathrm{i}}{2\pi c_3 c_2}}\exp\left\{\frac{(z_\mathrm{i}/c_2 + d_1 - z_\mathrm{f})^2}{2\mathrm{i}c_3} + \frac{\mathrm{i}c_1}{2}z_\mathrm{i}^2 + \mathrm{i}d_2 z_\mathrm{f} + \mathrm{i}d_3\right\}\cdot \quad (28\mathrm{a})$$

Where $c_1 = c_1(t,t)$ and similarly for the other c and d (for a definition see below).

In the other directions we have one-dimensional free particle problems with corresponding propagators

$$K_x(x_\mathrm{f},x_\mathrm{i},t) = \sqrt{\frac{1}{2\pi\mathrm{i}t}}\exp\left\{-\frac{(x_\mathrm{i}-x_\mathrm{f})^2}{2\mathrm{i}t}\right\}, \quad (28\mathrm{b})$$

$$K_y(y_\mathrm{f},y_\mathrm{i},t) = \sqrt{\frac{1}{2\pi\mathrm{i}t}}\exp\left\{-\frac{(y_\mathrm{i}-y_\mathrm{f})^2}{2\mathrm{i}t}\right\}\cdot \quad (28\mathrm{c})$$

The full propagator is the product of the three ones given above, *i.e.*

$$\tilde{K}^{\gamma,\beta}(\mathbf{r}_\mathrm{f},\mathbf{r}_\mathrm{i},t) = C(t)K_x(x_\mathrm{f},x_\mathrm{i},t)\,K_y(y_\mathrm{f},y_\mathrm{i},t)\,K_z^{\gamma,\beta}(z_\mathrm{f},z_\mathrm{i},t). \quad (29)$$

Now we proceed to the definition of the parameters given in (28a).

At first we set

$$f(t,\rho) = -\frac{\mathrm{i}}{\sqrt{V}}el(\omega)\frac{\mathrm{e}^{\mathrm{i}\omega\rho}}{\mathrm{e}^{\mathrm{i}\omega t}-1}, \quad (30\mathrm{a})$$

$$W(t,\rho) = -2\frac{1}{V}e^2l^2(\omega)\nu(t,\rho). \quad (30\mathrm{b})$$

As proved by Lo [17] if we suppose that $F(t,\rho)$ obeys the differential equation

$$\frac{\mathrm{d}^2F(t,\rho)}{\mathrm{d}\rho^2} + W(t,\rho)F(t,\rho) = 0 \quad (31)$$

with the initial condition $F'(t,0) = 0$, then c are given as

$$c_1(t,\rho) = \frac{\partial}{\partial\rho}\ln(F(t,\rho)), \quad (32\mathrm{a})$$

$$c_2(t,\rho) = \left|\frac{F(t,\rho)}{F(t,0)}\right|, \quad (32\mathrm{b})$$

$$c_3(t,\rho) = -F(t,0)^2\int_0^\rho \frac{\mathrm{d}u}{F(t,u)^2}\cdot \quad (32\mathrm{c})$$

In fact

$$W(t,\rho) = -\frac{4\pi\alpha}{V} - \frac{4\pi\alpha}{V}\csc\left(\frac{\omega t}{2}\right)\cos\left(\omega\rho - \frac{\omega t}{2} - \frac{\pi}{2}\right) \quad (30\mathrm{b}')$$

in dimensionless units.

Then equation (31) has as solutions Mathieu functions [20] and consequently we have the final results

see equations (33a, 33b, 33c) below

$$c_1(t,\rho) = \frac{\omega}{2}\frac{\text{MathieuC Prime}(a,q,z) + FF(t)\text{MathieuS Prime}(a,q,z)}{\text{MathieuC}(a,q,z) + FF(t)\text{MathieuS}(a,q,z)} \quad (33\mathrm{a})$$

$$c_2(t,\rho) = \left|\frac{\text{MathieuC}(a,q,z) + FF(t)\text{MathieuS}(a,q,z)}{\text{MathieuC}\left(a,q,\frac{\omega t}{4}+\frac{\pi}{4}\right) - FF(t)\text{MathieuS}\left(a,q,\frac{\omega t}{4}+\frac{\pi}{4}\right)}\right| \quad (33\mathrm{b})$$

$$\text{and}\quad c_3(t,\rho') = -\int_0^{\rho'}\frac{\left(\text{MathieuC}\left(a,q,\frac{\omega t}{4}+\frac{\pi}{4}\right) - FF(t)\text{MathieuS}\left(a,q,\frac{\omega t}{4}+\frac{\pi}{4}\right)\right)^2}{\left(\text{MathieuC}(a,q,z) + FF(t)\text{MathieuS}(a,q,z)\right)^2}\mathrm{d}\rho \quad (33\mathrm{c})$$

where

$$z = \frac{\omega\rho}{2} - \frac{\omega t}{4} - \frac{\pi}{4} \tag{33d}$$

$$FF(t) = \frac{\text{MathieuC Prime}\left(a, q, \frac{\omega t}{4} + \frac{\pi}{4}\right)}{\text{MathieuS Prime}\left(a, q, \frac{\omega t}{4} + \frac{\pi}{4}\right)} \tag{33e}$$

$$a = -\frac{4\pi\alpha}{\omega^2 V} \tag{33f}$$

and

$$q = \frac{2\pi\alpha}{\omega^2 V}\csc\left(\frac{\omega t}{2}\right)\cdot \tag{33g}$$

We have used the symbols of Mathematica [20] for the Mathieu functions which appear above, as in the present case we have an initial value problem with fixed a and q.

The d parameters are given as

$$d_1(t,\rho) = \gamma^* d_1'(t,\rho) + \beta d_1''(t,\rho) \tag{34a}$$

where

$$d_1'(t,\rho) = \int_0^\rho f(t,u)c_3(t,u)c_2(t,u)\mathrm{d}u, \tag{34b}$$

$$d_1''(t,\rho) = -\int_0^\rho f^*(t,u)c_3(t,u)c_2(t,u)\mathrm{d}u, \tag{34c}$$

$$d_2(t,\rho) = \gamma^* d_2'(t,\rho) + \beta d_2''(t,\rho), \tag{35a}$$

where

$$d_2'(t,\rho) = \int_0^\rho f(t,u)c_2(t,u)\mathrm{d}u, \tag{35b}$$

$$d_2''(t,\rho) = -\int_0^\rho f^*(t,u)c_2(t,u)\mathrm{d}u, \tag{35c}$$

and

$$d_3(t,\rho) = \gamma^{*2} d_3'(t,\rho) + \beta^2 d_3''(t,\rho) + \gamma^*\beta d_3'''(t,\rho), \tag{36a}$$

where

$$d_3'(t,\rho) = \int_0^\rho f(t,u)c_2(t,u)d_1'(t,u)\mathrm{d}u, \tag{36b}$$

$$d_3''(t,\rho) = -\int_0^\rho f^*(t,u)c_2(t,u)d_1''(t,u)\mathrm{d}u, \tag{36c}$$

$$d_3'''(t,\rho) = \int_0^\rho \big(f(t,u)d_1''(t,u) - f^*(t,u)d_1'(t,u)\big)c_2(t,u)\mathrm{d}u. \tag{36d}$$

3 Application to wavepacket propagation

Now we consider that the state of our charged particle has been initially prepared to be in the form of the Gaussian wavepacket

$$\Phi(z) = \left(2\pi\sigma^2\right)^{-1/4}\exp\left[-\frac{1}{4\sigma^2}z^2\right]\cdot \tag{37}$$

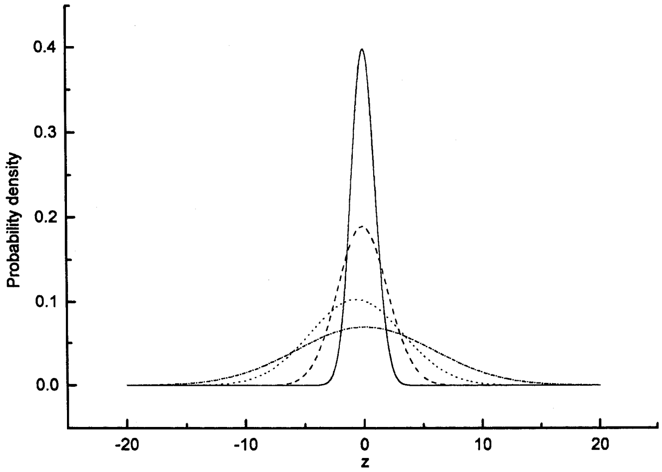

Fig. 1. Wavepacket probability density as a function of the z-coordinate for different times. Solid curve: $t = 0$. Dashed curve: $t = \pi/\omega$. Dotted curve: $t = 2\pi/\omega$. Dash-dotted curve: $t = 3\pi/\omega$. The following parameters have been set: $\omega = 5.0$ a.u., $V = 10^6$ a.u., $\beta = \gamma = 10^6$, $\sigma = 1.0$.

Such a state locates the particle with a width of σ at position 0.

The evolving wave function in the z-direction is obtained using the propagator (28a) as

$$\Psi(z,t) = \int K_z^{\gamma,\beta}(z,z',t)\Phi(z')\mathrm{d}z'. \tag{38}$$

Obviously the final wave function depends on γ and β. In the other directions we suppose free propagation.

On performing the integration we obtain

$$\Psi(z,t) = \frac{1}{\left(2\pi\sigma^2\right)^{1/4}\sqrt{\frac{1}{c_2} - c_1c_2c_3 - \frac{\mathrm{i}c_2c_3}{2\sigma^2}}} \times \exp\left[\begin{array}{l} -\frac{\mathrm{i}\sigma^2\left(d_1 - z\right)^2/c_2^2}{c_3\left(\mathrm{i}c_3 + 2c_1c_3\sigma^2 - 2\sigma^2/c_2^2\right)} \\ -\frac{2\mathrm{i}z^2}{c_3} + \frac{\mathrm{i}d_1z}{c_3} + \mathrm{i}d_2z - \frac{2\mathrm{i}d_1^2}{c_3} + \mathrm{i}d_3 \end{array}\right]. \tag{39}$$

We have assumed that $c_1 = c_1(t,t)$ and similarly for the others c and d.

In Figure 1 we give $|\Psi(z,t)|^2$ as a function of z for various times. We observe increasing spread and a small drift of the maximum of the distribution with time.

4 Conclusions

In the present paper we studied a model Hamiltonian that describes the interaction of radiation with electrons in a cavity, extracted the corresponding propagator in a closed form and considered the time evolution of an initially Gaussian wavepacket prepared in a cavity. As Mathieu functions are involved in the propagator, from the point of view of mathematical functions involved radiative processes in a cavity appear as considerably different from that in free space. Our results depend on the cavity volume and it is expected that for very large volumes they should approach free space results.

The present model is simple and tractable and gives new aspects of radiative processes in cavities. We intend to apply it in the study of atoms in strong fields.

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